647510 **BSc4MatC401x** **Seat No :________**

B.Sc. Semester - 4 (*CBCS*) Examination
March/April-2023 [NEW COURSE]
Mathematics-M401(Core)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q.1(A) Answer the following questions. (Any Two) (10)

1. State and prove Cauchy's first theorem on limits.
2. Prove that the sequence $\{S_n\}$ defined by $S_1 = \sqrt{7}, S_{n+1} = \sqrt{7 + S_n}$ converges to the positive root of the equation $x^2 - x - 7 = 0$.
3. Define: Convergence of sequence. Prove that every convergent sequence has unique limit.

Q.1(B) Answer the following question. (Any One) (04)

1. Show that $\lim_{n\to\infty} \left[\frac{1}{\sqrt{n}} + \frac{1}{\sqrt{n+1}} + \frac{1}{\sqrt{n+2}} + \cdots + \frac{1}{\sqrt{2n}}\right]$ does not exists.
2. If $\lim_{n\to\infty} a_n = a$ and $a_n \geq 0, \forall\, n \in N$ then prove that $a \geq 0$.

Q.2(A) Answer the following questions. (Any Two) (10)

1. Discuss the convergence of $\frac{1}{2\sqrt{1}} + \frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \frac{x^6}{5\sqrt{4}} + \cdots$
2. Discuss the convergence of $1 + \frac{3}{7}x + \frac{3\cdot6}{7\cdot10}x^2 + \frac{3\cdot6\cdot9}{7\cdot10\cdot13}x^3 + \cdots$ using Raabe's test.
3. Test the convergence or divergence of the series
   $\frac{\sqrt{2}-1}{1} + \frac{\sqrt{3}-\sqrt{2}}{2} + \frac{\sqrt{4}-\sqrt{3}}{3} + \ldots$

Q.2(B) Answer the following question. (Any One) (04)

1. Define: Absolutely convergent series. Show that the series
   $\sum(-1)^n \cdot \frac{n}{n^3+1}$ is absolutely convergent.
2. Find the radius of convergence and interval of convergence of the series
   $5x + 5^2x^2 + 5^3x^3 + \cdots$

Q.3(A) Answer the following questions. (Any Two) (10)

1. If $T: U \to V$ is any linear transformation then prove that R_T is a subspace of V and N_T is a subspace of U.
2. Verify rank nullity theorem for given linear transformation
   $T: R^4 \to R^3, T(e_1) = (1,1,1), T(e_2) = (1,-1,1), T(e_3) = (1,0,0), T(e_4) = (1,0,1)$.
3. Show that $T: R^4 \to R^4, T(a,b,c,d) = (a-b, b-c, c-d, d)$ is non singular and find T^{-1}.

Q.3(B) Answer the following question. (Any One) (04)

1. If $T: R^2 \to R^2, T(x,y) = (x cos\theta + y sin\theta, -x sin\theta + y cos\theta); \quad \forall (x,y) \in R^2$ and B_1, B_2 are standard basis of R^2 then find $[T: B_1, B_2]$.
2. Prove that composition of two linear transformation is again a linear transformation.

Q.4(A) Answer the following questions. (Any Two) (10)

1. Let V be an inner product space and $u, v \in V$. Prove that:
   $|u \cdot v| = \|u\|\|v\| \Leftrightarrow u$ and v are linearly dependent.
2. Using Gram-Schmidt orthogonalization process, obtain orthonormal basis of R^3 with usual inner product from the basis $\{(1,0,1), (1,0,-1), (0,3,4)\}$.
3. Let $C[0,1]$ be the vector space of all continues real valued functions defined on $[0,1]$ then show that: $\forall f, g \in C[0,1]$, $f \cdot g = \int_0^1 f(t)g(t)dt$ defines an inner product in $C[0,1]$.

Q.4(B) Answer the following question. (Any One) (04)

1. Let V be an inner product space and $u, v \in V$. If for every $w \in V,\ u \cdot w = v \cdot w$ then show that $u = v$.
2. Define: **(i)** Orthogonal and orthonormal set of vectors.
   (ii) Orthogonal and orthonormal basis.

Q.5(A) Answer the following questions. (Any Two) (10)

1. Show that radius of curvature at any point on the cardioid
   $r = a(1 - cos\theta)$ is $\frac{2}{3}\sqrt{2ar}$.
2. If $y = mx + c$ is an asymptote to the curve $y = f(x)$, prove that
   $m = \lim_{x \to \pm\infty} \left(\frac{y}{x}\right)$ and $c = \lim_{x \to \pm\infty} (y - mx)$.
3. Find multiple points of the curve $x^3 + y^3 - 3x^2 - 3xy + 3x + 3y - 1 = 0$ and determine its type.

Q.5(B) Answer the following question. (Any One) (04)

1. Find all the asymptotes of the curve $x^3 + y^3 - 3axy = 0$.
2. Find the interval of values of x for which the curve
   $y = x^4 - 6x^3 + 12x^2 + 5x + 7$ is concave upwards or downwards.
   Find also the points of inflexion in each case.

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